# A Free Form Feature Taxonomy

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**Abstract**

*In this paper the notion of* free form feature *for aesthetic design is presented. The design of industrial products constituted by free form surfaces is done by using CAD systems representing curves and surfaces by means of NURBS functions, which are usually defined by low level entities that are not intuitive and require some knowledge of the mathematical language. Similarly to the feature-based approach adopted by CAD systems for classical mechanical design, a set of high level modelling entities which provides commonly performed shape modifications has been identified. Particularly, the paper suggests a classification of the so-called* detail features *for an aesthetic and/or functional characterization of predefined free form surfaces. Feature types are formally described by means of an analytical definition of the surface modification through deformation and elimination laws. A topological classification is then given according to the application domain of such laws. A further sub-classification of morphological types is then suggested according to geometric properties of weak convexity and concavity for the resulting modified shape, leading to a taxonomy of simple free form features meaningful for aesthetic design.*

## 1. Introduction

In a very competitive world wide market, where product life time is shorter and shorter, quality and cost are not necessarily the only important success key elements. On the contrary also the aesthetic aspect is becoming fundamental for the customers. Since on the market it is possible to find products similar in quality and price, but differing only for some shape elements, the choice of the customer is strongly conditioned by the colour and the aesthetics of the product. In addition, the availability of new materials and production tools make possible the fabrication of very complex shapes, thus providing a greater freedom to the designers creativity.

However, shortening the time and cost for the development of new products still remains an important aspect. To this aim, new computer-aided tools have been developed to support the styling activity providing high quality rendering and animation facilities for simulation and evaluation purposes in order to reduce the number of physical prototypes, thus reducing the design costs. Generally these systems can represent complex-shaped objects by means of NURBS (NonUniform Rational B-Splines) [1, 2] surfaces, which are defined in terms of low level entities such as control points, knots and weights. The main limits of these systems are related to the supplied modelling tools, which normally use the surface control points for creating and modifying the surfaces, thus requiring a certain mathematical knowledge and experience in knowing how to modify these points in order to obtain the desired surface modifications.

In addition to this, despite the fact that similar products in the market slightly differ from each other by means of small details, current CAD/CAS systems do not support the fast generation of shape alternatives starting from already existing models. Changing a small detail sometimes is really difficult, and it can cost more than re-creating the entire model. This is still due to the fact that the modelling activity is based on low level geometric elements that do not preserve the design intent of the stylist.

In order to overcome the above limits, some modelling tools closer to the stylist’s way of thinking are currently being developed and new ones need to be provided. The Brite-Euram Project N. BE96-3579 called FIORES [3, 4] (Formalization and Integration of an Optimized Reverse Engineering Styling Workflow) is focusing on this problem. In particular, the two main aspects considered by the project, related to the styling activity, are: identification of new modelling tools for direct shape modification and *engineering in reverse* techniques for surface manipulation. The first point is more aimed at making faster the model creation and allowing the existing models re-use, it takes advantage of the feature technology, well established in the mechanical environment. The second one is strictly related to shorten the frustrating loop consisting of surface modification and aesthetic evaluation of the created product model. While in the past this evaluation was possible only on the clay models, at present it is commonly done on the digital model by means of 3D shading for simulating the real object, and is based on some *evaluation* lines that give the expert designer an idea of the aesthetic quality of the object. These lines range from section curves to reflection lines up to special curvature lines. It is not unique how these lines patterns are interpreted, i.e. when they indicate a *good* or a *bad* surface. The designer rather learns this from the practical work with samples (in fact, very commonly CAS operators have good experience in clay modelling) and, therefore, he needs to learn how to modify the object surfaces in such a way that the derived evaluation lines result *good*. With the *engineering in reverse* (EiR) approach adopted by FIORES the modification loop can be avoided: the designer can directly specify the target evaluation line, which he considers better than the actual, together with additional constraints, such as the modifiable area, and then the system from this information returns a surface with the given evaluation line. As a matter of fact, in order to be sure of having a solution, an optimization approach is considered that computes the best solution, i.e. the solution that under a certain criterion is as close as possible to the desired target. However, this automatic process requires the final evaluation by the stylist to check whether the model is good or not. In order to solve these problems, the FIORES consortium assembles complementary and common skills of the styling application as well as fundamental research and software developers in the in the area of CAD/CAS.

The present paper deals with the direct modelling approach. As a preliminary activity, the identification of modelling tools, according to a feature-based approach, requires an analysis of the various classes of local shapes (free form features) that can be generated/recognized over a complex-shaped product. Such analysis allows to single out the sets of geometric and topological elements, which characterize a local shape over the overall surface, that can be used as a complex modelling entity created/modified through a set of high level parameters. With that purpose, a formal definition of shape classes over a free form surface is here given.

The paper is organized as follows. Chapter 2 introduces the concept of free form feature and the differences between mechanical and aesthetic features. In Chapter 3, the working method for styling design is analysed. According to it, two types of features are distinguished in Chapter 4: structural features and detail features. Focusing on detail features, the concept of characterization law defining a general free form feature, based on surface modification by deformation and elimination of parts, is described in Chapter 5. Chapter 6 suggests a possible formalization of features by means of a deformation function applied on the surface to be characterized. Various topological classes of deformations are successively analysed. By distinguishing extrusions from intrusions, a definition of simple morphological feature classes by deformation is then given, depending on properties of weak convexity and concavity of the feature surface. In Chapter 7 features obtained by elimination of parts are described and, coherently with the approach followed for deformations, various topological classes of sharp and refined cuts are analysed. Finally, for application purposes, a list of possible significant geometric parameters, defining the shape of the considered feature classes, is briefly discussed in Chapter 8.

## 2. The Form Feature concept

Form features have been introduced in mechanical environments as the key elements for associating specific functional meaning to groups of geometric elements (faces, edges and vertices), thus offering the advantage of treating sets of elements as single entities [5]. Such entities are much more meaningful for application than the low level constitutive elements and are manipulated by means of a limited number of significant parameters. It is quite clear that using features as design primitives improves the efficiency in creating the product model, in considering alternative solutions and shortens the time required for model changes. While the concept of feature has been mainly investigated in mechanical environments [6, 7, 8], only few activities have been done for free form modelling [9, 10, 11, 12, 13]. This is mainly due to the fact that classical mechanical parts are defined by canonical geometric shapes, which can be easily classified, and the association between shape and function can be better identified. In Fig. 1 some types of form features used in mechanical design are shown. In order to define a feature-based design system, it is necessary to identify the set of primitives (features) that are useful for the selected application domain. Trying to define a concept of form feature which can be used in styling activities, we intend to consider parts of an object whose function corresponds to an aesthetic meaning. Considering this, not all the types of features defined for mechanical CAD systems can be directly used in the frame of aesthetic design. In this case the designer needs a certain creativity, a certain freedom of expression that takes into account personal tastes, visual impressions, fashions, etc. which cannot be achieved by means of the rigid shapes constituting mechanical features. What can be recovered is in any case a methodology in the modelling activity: features as complex high level entities that allow a fast creation and modification of the geometric model [5, 14]. In both contexts (mechanical and aesthetic design) what makes one feature different from another is the set of the geometric and topological elements that define the shape and the appearance of the feature, and also the type of interaction with the remaining geometry of the object. A form feature is what makes ”different” (*functionally* in the mechanical design, and *aesthetically* in the styling activities) two objects that were ini-

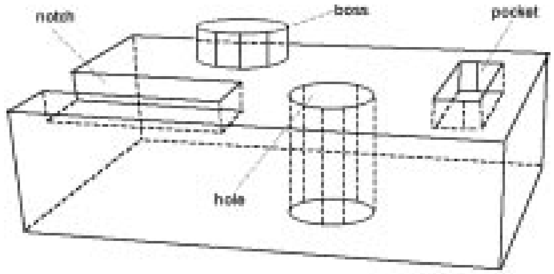


**Figure 1:** *Mechanical form features*

tially defined by the same overall geometry (for example, the same surface or the same shape as support).

## 3. Styling activity

In order to identify the set of features meaningful for aesthetic design, the working methods of stylists have been analysed. The considered questions are: what are the common ideas followed by the designers when they sketch a shape? What are the criteria used? Observing how designers define the aspect of the products, it appears that the definition of a shape is performed in two logical steps:

- Overall shape definition, providing the product global effect;
- Details definition, originating the complete final shape of the product.

This two steps approach is used both for hand sketching and for CAS/CAD modelling. In particular, in the computer assisted activity, in the first phase the designer creates the object's enveloping surfaces starting from some essential curves that bound the overall shape. These curves can be thought as the skeleton of the product model, that is what would be drawn by hand for visualizing the object, and correspond not only to structural lines, like object profiles and selected sections, but also to significant lines strongly affecting the product shape, i.e. the so-called *character lines* [15, 16]. The quality and the aesthetics of such curves is quite important since they are used for creating the surfaces enveloping the product, thus the global product impression is strongly dependent on their shape and quality. Thus, many checks are made on them, with different tools, since their modification when the product model is almost complete is very cumbersome, requiring the manual modification of most of the created surfaces. In Fig. 2, as an example of quality analysis of the structural curves, the horizontal compression for emphasizing the curvature defects is depicted. In the second step, according to the *a posteriori* modelling approach, the details characterizing the object functionally and aesthetically are added. This corresponds to modifications of the surfaces previously created possibly with the generation of new surfaces. While specific contours can be really existing curves

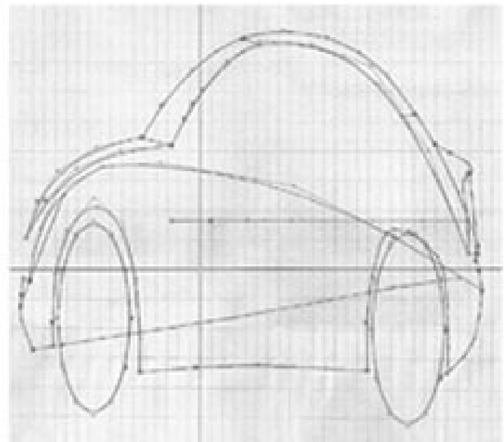

**Figure 2:** *Compressed structural curves of a car model (by Pininfarina*


**Figure 3:** *Barchetta (by Fiat)*

when creating the surface based model of the product, most of the character lines are *virtual* (i.e. only perceived) lines, and they are obtained as set of surfaces or through particular relations among surfaces (see Fig. 3).

## 4. Features in styling

According to the different phases of the design activity in styling, and keeping in mind the requirements that a feature has to satisfy, it is possible to identify two categories of form features in the aesthetic design [16]:

1. **Structural features**, created in the preliminary phase of design. They are the structural entities used for defining the surfaces constituting the product, thus having an important aesthetic impact.
2. **Detail features**, created in the second modelling phase. They are applied on a surface for adding aesthetic and

functional details and for enforcing the visual effects of character lines.

Features of the first kind serve to group connected sets of curves according to their intrinsic meaning (i.e. contours and character lines) to treat them as a unique entity and to maintain the relationships with the corresponding derived surfaces. They constitute the basis for an easier and more efficient successive modification of the whole product by varying the shapes of the structural elements.

The current paper focuses on detail features that are used in the a posteriori modelling phase. According to the approach that considers detail form features as obtained by local modifications of a given free form surface, in the following a formal definition of features types is presented, based on an analytical definition of surface modification through deformation and elimination laws, leading to a mathematical classification of feature types based on topological properties. A further sub-classification into morphological types according to properties of convexity/concavity will lead to a taxonomy of simple free form features significant for aesthetic purposes.

Examples of structural and detail features are shown in Fig. 18 (see color plates), where some lines, which characterize the shape aspect of the car body of the Nautilus designed by Pininfarina, are highlighted with different colors. In particular, light red represents character lines, cyan and green represent respectively profiles and other sections. Different detail features are highlighted in yellow and light green.

## 5. Detail features

Detail features correspond to local modifications of given free-form surfaces to create objects having complex shapes. These features have to cover in a sufficiently exhaustive way the three possible operations that can be done on a local part of a surface:

- adding a region,
- substituting a region,
- removing a region.

While the first operation can be performed by union operations on already defined shapes, or through the use of modelling operations such as fillets and blendings that can be considered also as features (not treated here since already well known), the latter two operations can be associated to the two-fold idea of deformation and elimination.

In both cases, given a free-form surface, a new surface is generated from the former. The difference is the following: the operation of deformation transforms points into points, while in the operation of elimination points are removed (See Fig. 4). We define then as *characterization law* the transformation criterion that is applied to a surface for obtaining the modified surface corresponding to the detail form feature. If no parts are removed, the characterization law is

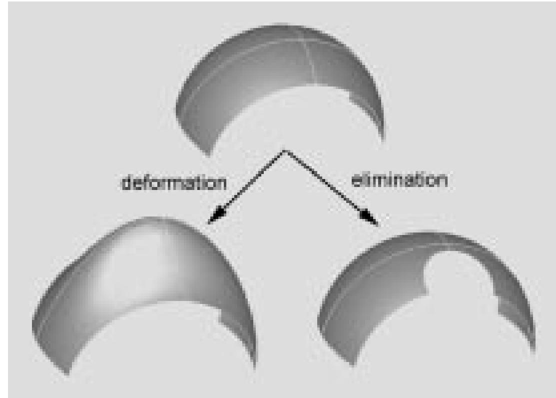


**Figure 4:** *Deformation and elimination operations applied on a surface*

called *deformation law* and it can be seen as a mapping function that transforms points of the primary surface into points of the modified surface. In the case of features corresponding to removal of a surface region, the characterization law is indicated as *elimination law* and it could simply correspond to a trim operation. The characterization law could be implemented as a sequence of predefined modelling operations or by using other deformation criteria like the ones based on mechanical approach [10, 17].

We will indicate as *primary surface* the patch, or a set of adjacent patches, to be modified by a detail feature and *characterized surface* the resulting transformed surface. Considering a feature as a surface deformation or elimination, it is important to define some key elements:

- the area of the surface that has to be modified;
- the type of characterization to be performed;
- which is the behaviour along the contacts between the modified and the unmodified parts.

The proposed feature classification, given in the following chapters, is based on the information defining the characterization law, in particular on the topological properties of its application domain, in the following called *influence area*, i.e. the primary surface region to be modified, and on the morphological aspects of the resulting shape, i.e. the image of the deformation.

## 6. Features by deformation

### 6.1. Deformation law

Broadly speaking, from the above assumption, given an object **O** obtained from the enveloping surfaces created in the first styling phase, the final model **O'** can be seen as a new set of surfaces obtained from **O** by the application of a sequence of local deformations and eliminations.

In order to define the conditions and restrictions for the specification of deformations giving raise to meaningful free form features, some general definitions will be given in the following [1, 18].

Let us consider each connected surface $S$ of the object **O** as the current primary surface to be modified. It is assumed that $S$ is union of regular patches of class $C^2$.

**Def. 1** *Given a primary surface S and a surface region $A_I \subset S$, each function $\delta : S \to \Re^3$ is said to be a deformation law with influence area $A_I$, if it can be defined as*

$$\delta = \begin{cases} i & on \;\; S - A_I \\ \delta_h & on \;\; A_h \;, \forall h = 1, 2, \ldots, n \end{cases}$$

*for a finite decomposition $\{A_h\}_{h=1,2,\ldots,n}$ of disjoint surface regions $A_h \subseteq A_I$, such that $\bigcup_{h=1}^{n} A_h = A_I$ satisfying the following properties:*

1. *i is the identity function;*
2. *$\delta_h$ is an omeomorphism of $A_h$;*
3. *$\delta_h(A_h) \cap \delta_k(A_k) = \emptyset \; \forall h \neq k$;*
4. *$P \neq \delta(P), \forall P \in \overset{\circ}{A_h}$ .*

It is then possible to define a deformation feature as the resulting shape of the deformation. Precisely:

**Def. 2** *Given a primary surface S and a deformation law $\delta : S \to \Re^3$ with influence area $A_I \subset S$, then the effective modified surface region $F_\delta = \bigcup_{h=1}^{n} \delta(A_h)$ is called free form δ-feature (δ-FFF).*

The characterization of the new surface is then given by the global image $S' = \delta(S) = (S - A_I) \cup \bigcup_{h=1}^{n} \delta(A_h)$, since the overall aesthetic impact is not only given by the δ-feature itself but also by its position in the surface and the way it interacts with the remaining unmodified geometry. The influence area $A_I$ is the region of the primary surface $S$ that will be really modified by the insertion of the free form feature, except for a finite number of curves or isolated points at most, while outside it points remain fixed. It can be composed of more patches and, in the induced topology over $S$ from the euclidean space $\Re^3$, it can be open, closed or neither open nor closed. The above definition expresses formally the idea of a general deformation as union of a finite number of omeomorphisms, so that the image is still a surface maintaining locally the topological properties of the primary surface thanks to the local omeomorphism but not necessarily globally, since possible breaks of continuity over some surface curves can be allowed, changing the topological type of connection. Such a distinction can be defined recurring to the well-known geometric continuity conditions $G^k$:

**Def. 3** δ *is said deformation with break if at least a curve $\gamma \subset A_I$, called break-curve, exists such that a $G^{-1}$ condition is satisfied for two surface regions in $S'$ obtained through the transformation δ of two surface regions in S adjacent to γ. Otherwise, δ is said elastic deformation.*

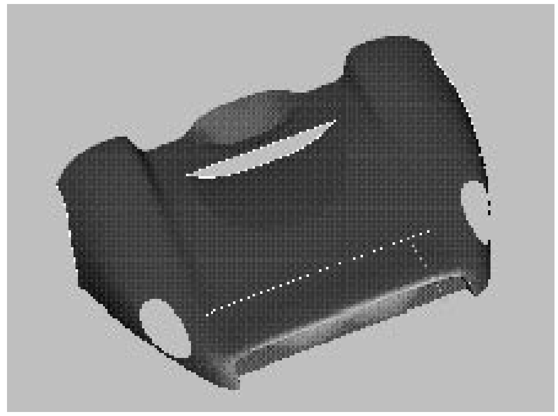

**Figure 5:** *Example of deformation with break*

In case of deformations with break, it is assumed that possible break-curves lie on the influence boundary and not in the interior of the influence area. This is not a restrictive hypothesis since an internal break can in any case be generated by joining two successive separate deformations having contiguous influence areas adjacent to the break curve contained on both the corresponding influence boundaries.

A δ-FFF, representing a type of deformation with break, is shown in Fig. 5

Because of the above generalization with respect to a single global omeomorphism, the characterized surface can result even disconnected. Since surfaces have to represent in any case shapes of real existing objects, it is assumed for the following that any connected component cannot degenerate into isolated points, or curves.

### 6.2. Topological classes of deformations

According to the topological type of connection of a neighbourood of the influence area in the induced topology over S, it is possible to distinguish three classes of possible deformations:

**Def. 4** δ *is said (a) border deformation, (b) internal deformation, (c) channel deformation if a surface neighbourhood $J_{A_I}^S \subseteq S$ of $A_I$ exists such that $J_{A_I}^S - A_I$ is respectively (a) simply connected, (b) connected but not simply connected, (c) disconnected.*
*The corresponding free form δ-feature is said (a) border δ-FFF (Bδ-FFF), (b) internal δ-FFF (Iδ-FFF), (c) channel δ-FFF (Cδ-FFF).*

This means that for an internal deformation the influence area does not intersect the surface boundary, differently from border deformations. Channels can be either internal or crossing the surface boundary. In Fig. 6 an internal, a

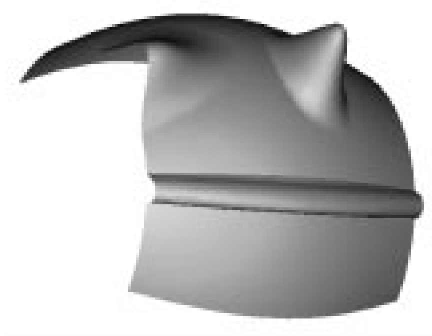

**Figure 6:** *Types of deformations*

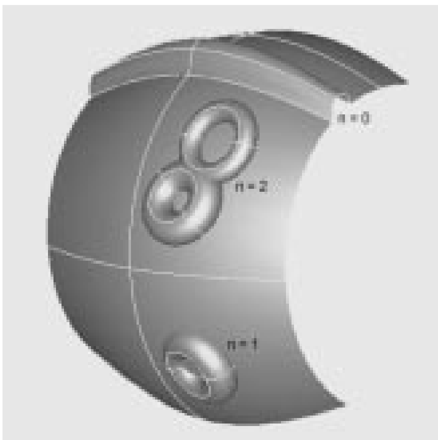


**Figure 7:** *Examples of n-channels*

channel and a border deformation are respectively shown. Depending on the disconnection type of the unmodified region, several types of Cδ-FFFs can be distinguished. By indicating as *δ-isle* each possible connected component of $S - A_I$ whose boundary is a closed curve internal to another closed curve of the influence boundary, a distinction among channels can be given according to the number of existing isles:

**Def. 5** *A channel δ-FFF giving raise to n δ-isles, for* $n = 0, 1, 2, \ldots$, *is called a n-channel.*

A 0-channel always intersects the surface boundary. See in Fig. 7 different types of n-channels for $n = 0, 1, 2$. Once a top level topological classification of the possible deformation features is defined, it is then possible to further classify them according to the morphological aspect of the surface resulting from the deformation itself.

### 6.3. Intrusions and extrusions

The assumed regularity for the parametrization of the surface allows to define a normal unit vector at each point for each patch $S_i$ composing $S = \cup_i S_i$ and, by right-handed rule, the corresponding orientation of each closed patch boundary. In order to formally distinguish an interior and an exterior for the surface, we consider only *oriented* surfaces [18], composed of *coherently* oriented patches (i.e. each curve belonging to the boundary of two adjacent patches has two opposite orientations with respect to the chosen parametrizations of the two involved patches). This is not a too restrictive hypothesis since coherence among patches can always be obtained possibly through re-parametrization. The *exterior* of the surface is then defined as its side corresponding to the region of space where all normal unit vectors point.

A general deformation gives raise to a surface where regions modified towards the exterior or the interior of the primary surface can exist. On this regard, given a deformation law δ over $S$ with influence area $A_I \subset S$ :

**Def. 6** δ *is said extrusion (intrusion) over* $B \subseteq A_I$ *if* $\forall P \in B$*, with* $P' = \delta(P)$ *, the oriented vector* $\vec{PP'}$ *is concordant (discordant) with the normal unit vector* $\hat{n}$ *at S in P, i.e.* $\hat{n} \cdot \vec{PP'} > 0 \, (\hat{n} \cdot \vec{PP'} < 0)$.

Because of the regularity of each patch in $S$ and $S'$, the domain $A_I$ can be decomposed in a finite number of open connected surface regions $\{B_{ext,h}\}_{h=1,2,\ldots,n_E}$ and $\{B_{int,k}\}_{k=1,2,\ldots,n_I}$ over which δ is pure extrusion, intrusion respectively. Each point $P \in A_I$ such that $P \in \bigcup_{h=1}^{n_E} \partial B_{ext,h} \cup \bigcup_{k=1}^{n_I} \partial B_{int,k}$ belongs to a break-curve or a curve of fixed points for δ. Therefore the general deformation $\delta : S \to \Re^3$ can be seen as

$$\delta = \begin{cases} i & \text{on S - } A_I \\ \delta_{ext,h} & \text{on } B_{ext,h}\,, \forall h = 1, 2, \ldots, n_E \\ \delta_{int,k} & \text{on } B_{int,k}\,, \forall k = 1, 2, \ldots, n_I \end{cases}$$

Where $\delta_{ext,h}$ and $\delta_{int,k}$ are omeomorphisms representing a pure extrusion and intrusion over $B_{ext,h}$ and $B_{int,k}$ respectively. In the following we will focus on elementary free form δ-features with δ pure extrusion and intrusion, since more complex deformations can be in any case obtained by joining a finite number of local extrusions and intrusions.

### 6.4. Detail features preserving the primary shape

Both for extrusion and intrusion the shape resulting from the deformation may preserve more or less the aspect of the original shape of the primary surface contained in the influence area.

A typical example is given by modifications where a part is obtained by a rigid translation of a portion of the influence area along a fixed direction. In this case the deformation law δ is a well-known analytic function in that portion. Precisely:

**Def. 7** *A δ-FFF, whose influence area is* $A_I \subset S$ *, is said a displacement δ-FFF if a surface region* $A_D \subseteq A_I$ *(called displacement area) exists such that* $\delta(P) = P + h\hat{v}, \forall P \in A_D$*, where* $\hat{v}$ *(called displacement direction) and h (called*

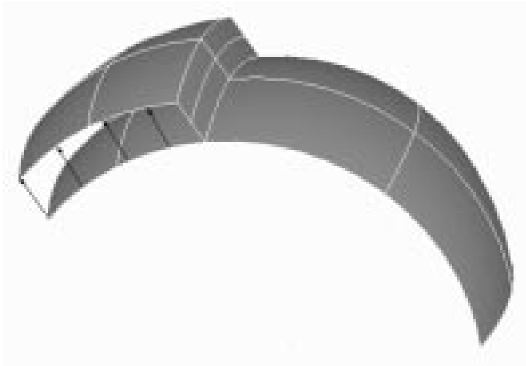

**Figure 8:** *Representation of an extrusion by displacement*

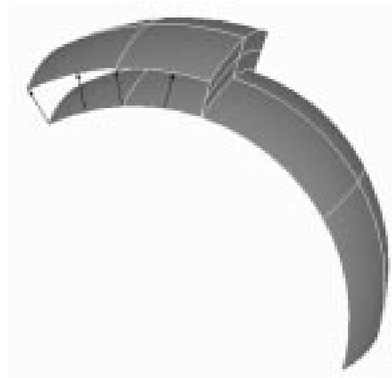

**Figure 9:** *Representation of an offset-up surface*

*displacement width) are respectively a fixed unit vector and a constant scalar.*
*If the displacement δ-FFF is an extrusion, it is called displacement-up, if it is an intrusion it is called displacement-down.*

In the area $A_I - A_D$ , where the deformation does not originate breaks, displacement features are supported by a surface that connects the unmodified region with the displaced part. Generally variations of curvature between the connecting surface and the displacement make a character line well singled out, separating the two parts. See Fig. 8 as an example of displacement δ-FFF.

It could also be possible to consider intrusions and extrusions created through more general rigid motions consisting in simultaneous rotations and translations of parts of the primary surface, still requiring additional surfaces connecting the displaced part with the unchanged region.

Another case of shape preservation in scale by dilatation or contraction of a region in the primary surface can be considered, and it can be defined as follows:

**Def. 8** *A δ-FFF, whose influence area is $A_I \subset S$, is said a offset δ-FFF if a surface region $A_O \subseteq A_I$ (called offset area) exists such that $\delta(P) = P + h\hat{n}_P$, $\forall P \in A_O$, where $\hat{n}_P$ is the normal unit vector at S in P and h is a constant scalar (called offset width).*
*If the offset δ-FFF is an extrusion (i.e. $h > 0$), it is called offset-up, if it is an intrusion ($h < 0$) it is called offset-down.*

For these features additional checks on the domain $A_O$ with respect to the specified width have to be considered. Depending on the situation, possible restrictions over $A_O$ might be needed in order to avoid self-enveloping surfaces. In this case $A_O$ has to be modified with a new one $A_O^*$, and a new δ should be considered for $A_O - A_O^*$ . Moreover, since breaks are produced by the offset when $A_O$ contains patches of $S$ joined with a $G^0$ continuity condition, additional blending or fillet operations should be considered for connecting the separated parts. Finally, in the remaining region $A_I - A_O$ offset δ-FFFs are supported by a surface connecting the unmodified region with the offset, possibly giving rise to the emergence of virtual lines well singled out, very important for the aesthetic effect. See Fig. 9 as an example of offset δ-FFF.

### 6.5. Convexity and concavity of a surface in parametric form

In this section, concepts about global and local convexity for an oriented and coherent surface given in parametric form are introduced as a basis for the specification of the suggested free form δ-feature taxonomy. This is done by considering a geometric parametric approach [19] that requires the analysis of the behaviour of planes passing trough the points of the surface.

Let $S$ be a piecewise regular connected surface having parametric representation $S(u,v)$, $(u,v) \in U$, where $U$ is an open set of $\Re^2$.

**Def. 9** *For each point $P_0 = S(u_0, v_0)$, we define global supporting plane of S in $P_0$ each possible plane π passing through $P_0$ such that $\forall P, Q \in S$, with $P \neq P_0$ and $Q \neq P_0$, the segment $[P,Q]$ does not intersect π , unless it is totally contained in π .*

The above property could be fullfilled only for a local neighbourhood of the point on the surface. That is:

**Def. 10** *For each point $P_0 = S(u_0, v_0)$, we define local supporting plane of S in $P_0$ each possible plane π passing through $P_0$ for which an open neighbourhood $U_0 \subseteq \Re^2$ of $(u_0, v_0)$ exists such that $\forall P, Q \in S_0 = \{S(u,v), (u,v) \in U \cap U_0\}$, with $P \neq P_0$ and $Q \neq P_0$ , the segment $[P,Q]$ does not intersect π, unless it is totally contained in π.*

Evidently, a global supporting plane is also a local supporting plane, but not viceversa. Besides, given $P_0 \in S$, if the tangent plane of $S$ in $P_0$ exists and it is a local supporting

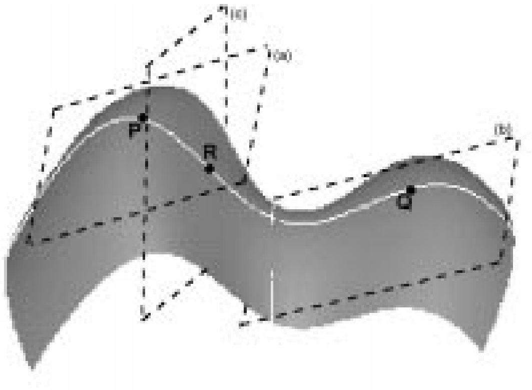


**Figure 10:** *Examples of global, local and neither global nor local supporting plane*

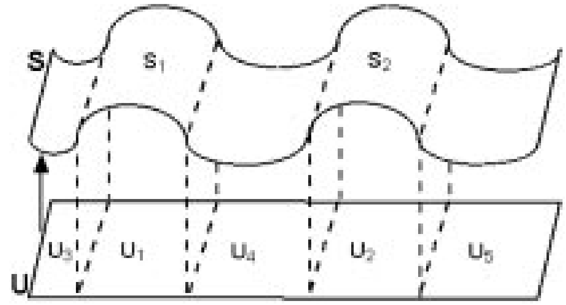


**Figure 11:** *Convexity and concavity of a surface*

plane in $P_0$, then it is the only one. If $P_0$ is a singular point, more local supporting planes of $S$ in $P_0$ can exist. If the surface $S$ is planar, evidently it coincides with its supporting plane for each point.

See in Fig. 10 plane (a) as an example of global supporting plane for the surface in $P$, plane (b) as a local but not global supporting plane for the surface in $Q$, and case (c) as an example of plane that is neither a global nor a local supporting plane in $R$. The previous definition of global and local supporting plane allows to extend the concept of convexity also for a surface given in a parametric form as generalization of the analogous well-known term from functional surfaces. Considering distinction among elliptic, parabolic and hyperbolic points over a surface [1, 18], it is clear that it is not possible to have local convexity (concavity) in a hyperbolic point. Besides, by considering an oriented surface whose patches have a $C^2$ regularity, for which an exterior and an interior is defined (as in our case), it is possible to specify a formal distinction between convexity and concavity for a surface region. Precisely:

**Def. 11** *Given a connected piecewise regular surface $S$ of class $C^2$, with $S = \{S(u,v),(u,v) \in U\}$, then $S$ is said convex (concave) on $V \subseteq U$ if $S_V = \{S(u,v),(u,v) \in V\}$ is such that:*

1. *no planar regions exist in $S_V$;*
2. *$\forall P \in S_V$ there exists a global supporting plane $\pi$ of $S_V$ in $P$;*
3. *$\forall P \in S_V, \exists Q \in S_V$, the vector $\vec{PQ}$ is such that $\vec{PQ} \cdot \hat{n_P} \leq 0$ (respectively $\vec{PQ} \cdot \hat{n_P} \geq 0$ ) where $\hat{n_P}$ is the normal unit vector at $S_V$ in $P$.*

*$S$ is said globally convex (globally concave) if it is convex (concave) on the whole parametric domain $U$. $S$ is said locally convex (locally concave) in $P_0 = S(u_0, v_0)$ if an open neighbourhood $U$ of $\Re^2$ exists such that $S$ is convex (concave) on $U \cap U_0$.*

Global convexity/concavity is also local, but not viceversa. Evidently, a planar region is neither convex nor concave, but it is well-known that neither convex nor concave non planar regions can exist (e.g. hyperbolic paraboloid). Besides, concavity implies convexity for the surface with inverted orientation (exterior and interior mutually changed) and viceversa. See in Fig. 11 an example of a surface that is convex on $U_1$ and $U_2$ and concave in $U_3$, $U_4$ and $U_5$, where $\partial(S_1 \cup S_2) - \partial(S)$ is a disconnected set of inflection lines.

### 6.6. Morphological classes of simple features

After having introduced concepts of global and local convexity for parametric surfaces, in this section a sub-classification will be given for internal, border and channel elastic extrusions and intrusions, by considering opportune properties of convexity/concavity of the corresponding free form δ-feature.

For an useful classification of aesthetic simple shapes, in which the above concepts of convexity/concavity are used in a more extended way, it is meaningful to consider also possible cases in which planar regions exist adjacent to convex (concave) parts but not to concave (convex, respectively) parts. On this regard, we will indicate as *convexity extension* (*concavity extension*) for a surface $S$ each possible planar region $S_P \subseteq S$ such that a convex (concave) region $S_1 \subseteq S$ exists for which $\partial S_P \cap \partial S_1$ contains at least a curve, while it is $\partial S_P \cap \partial S_2 = \emptyset$ for each concave (convex) region $S_2 \subseteq S$. Besides, a possible generalization of convexity (concavity) in weak sense requires to consider also points of local convexity (concavity) that can keep connection (e.g. surface curves of parabolic points) among strong convex (concave) regions. With regard to this, in the following, each connected union of a convex (concave) surface with all its convexity (concavity) extensions and all points of local convexity (concavity) will be indicated as *weakly convex* (*weakly concave*) region of $S$. Considering convex/concave parts (or weakly convex/concave, in case of existence of internal planar regions) of the shape of δ-FFF, significant regions of the influence area result then distinguished:

**Def. 12** *Given a δ-free form feature with influence area $A_I$,* *we define* weak convexity area *the region* $A_{CV} \subset A_I$ *such that* $\delta(A_{CV})$ *is the maximal union of weakly convex parts of* $\delta(A_I)$. *We define* weak concavity area *the region* $A_{CC} \subset A_I$ *such that* $\delta(A_{CC})$ *is the maximal union of weakly concave parts of* $\delta(A_I)$ .

A possible criterion for considering a local shape over a primary surface as complex is given by the number of weakly convex and concave connected parts of the influence area: the larger is the number of transition among weak convexity and concavity, the more complex the shape is. On this regard, given an arbitrary deformation law, by studying each single connected extrusion and intrusion, it is possible to consider in general δ-FFFs simple parts representing extrusions (intrusions) characterized by a weak convex (concave, respectively) region.

For aesthetic purposes, in the following suggested classification of simple morphological δ-feature classes, deformations are considered without any break. Therefore:

**Def. 13** *A δ-FFF, representing a pure elastic extrusion (intrusion respectively), is said* simple *if its weak convexity area (concavity area, respectively), is a non-empty connected region having the same topological type of the influence area. Otherwise it is said* compound.

Thus, for a simple δ-FFF it is important to distinguish the role of weakly convex and weakly concave regions inside the influence area:

**Def. 14** *The connected convex (or concave) region* $\delta(A_{CV})$ *(*$\delta(A_{CC})$ *, respectively) of a simple extrusion (intrusion) is called* kernel surface, *while the (possibly empty) remaining part* $\delta(A_I - A_{CC})$ *(* $\delta(A_I - A_{CV})$, *respectively) is called* transition surface.

Given the non-empty kernel surface defining a simple δ-feature, the transition surface will possibly exist surrounding the kernel totally or partially in the area where smoothing transition conditions between the kernel and the unmodified region, like $G^1$ or $G^2$ conditions, are required. According to the above definition, see in Fig. 12 an example of simple extrusion over a simply connected influence area. The above definition does not result too much restrictive since it is possible to consider complex shapes as the union of a finite number of compound extrusions and intrusions obtained by joining simple δ- features having only a connected kernel inside the area. Distinguishing the various topological classes of δ-FFFs, it is then possible to single out different simple shapes, particularly significant for aesthetic purposes. In the extrusive case:

**Def. 15** *A δ-free form feature obtained by a simple internal extrusion is said a* bump. *If it is obtained by a simple border extrusion, it is said a* step-up. *If it is obtained by a simple n-channel extrusion, it is said a* n-rib.

Analogously, in the intrusive case:

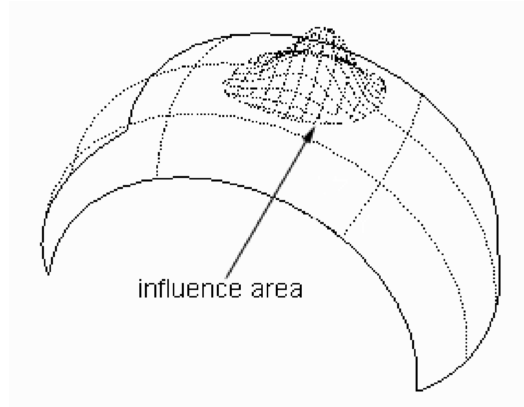


**Figure 12:** *A simple extrusion*

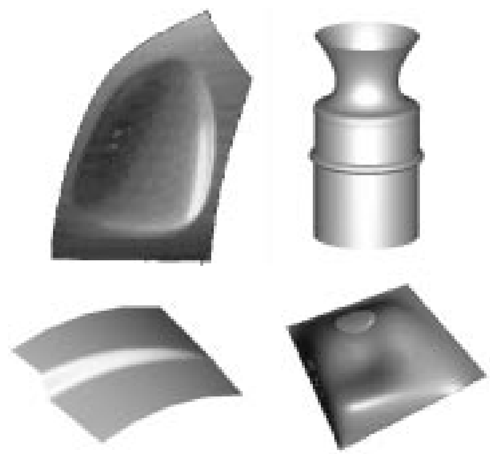

**Figure 13:** *Examples of deformation*

**Def. 16** *A δ-free form feature obtained by a simple internal intrusion is said a* cavity. *If it is obtained by a simple border intrusion, it is said a* step-down. *If it is obtained by a simple n-channel intrusion, it is said a* n-groove.

Inverting the exterior with the interior of the primary surface, evidently a bump is transformed in a cavity, a step-up becomes a step-down and a n-rib is changed into a n-groove. In Fig. 13 several types of simple free form δ-features are shown. According to Def. 16, they represent a cavity, a boss, a step, and a 0-rib, respectively.

## 7. Features by elimination

### 7.1. Sharp and finished cuts

Elimination features correspond to the removal of a surface region and always involve the use of a trim operation. Depending on the finishing operations performed around the trimmed area, it is possible to define some macro-categories.

**Figure 14:** *Examples of sharp and finished cuts*

A pure removal of parts can be expressed by the following boolean subtraction:

**Def. 17** *Given a primary surface $S$ and a connected surface region $A_T \subset S$, the transformation $S' = S - A_T$ is said a sharp cut. $A_T$ is called trimmed area and the corresponding $\partial A_T$ trim boundary.*

For aesthetic purposes, a part of the surface along the profile of the cut is usually smoothed. It is then possible to define a refined cut as the result of a initial sharp cut with a successive opportune local deformation around the trimmed area. Precisely:

**Def. 18** *Given a primary surface $S$, a finished cut is defined as $S' = \delta_T(S - A_T)$ where $A_T \subset S$ is a connected surface region and $\delta_T$ is a deformation law on $S - A_T$ whose influence area is a surface region $E_T \subset S - A_T$ such that $E_T \cup A_T$ is a surface neighbourhood of $\mathring{A}_T$. $E_T$ is then called finishing area of the cut.*
*Moreover, the finished cut is said totally finished if $\overline{E_T} \supset \partial A_T$, otherwise it is said partially finished.*

According to the intuitive idea, the finishing area of a totally finished cut completely surrounds the trim boundary. In Fig. 14, a sharp cut, a totally and a partially finished cut are respectively shown.

For aesthetic design a certain level of smoothness is usually required to the deformation law $\delta_T$: generally it is given as completely extrusive or intrusive, e.g. it could give raise to a fillet surface. The identification of rules for setting the refinement conditions for cuts requires a deep analysis in which aesthetic criteria have to be considered together with the optical effects due to particular bendings of the surface and feasibility restrictions depending on the material and the production technologies.

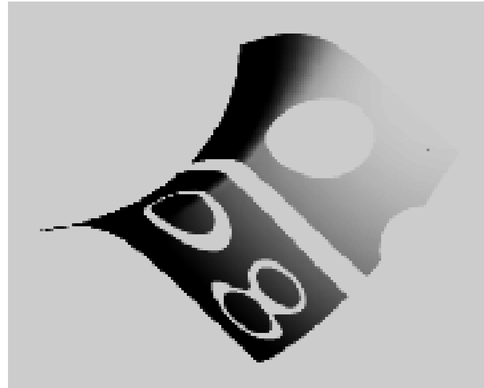

**Figure 15:** *Types of cuts*

To distinguish sharp and finished cuts from free form $\delta$-features, they are called *free form $\tau$-features* ($\tau$-FFFs), where $A_T$ and $E_T \cup A_T$ correspond to the influence area $A_I$ of the $\delta$-features, respectively.

### 7.2. Topological classes of cuts

Following the same approach used for the topological classification of deformations, depending on the connection properties of an opportune neighbourhood of the trimmed area in the induced surface topology, three classes of possible cuts are singled out:

**Def. 19** *A sharp or finished cut, with trimmed area $A_T$, is said (a) inlet (b) hole, (c) gap if a surface neighbourhood $J^S_{A_T} \subset S$ of $A_T$ exists such that $J^S_{A_T} - A_T$ is respectively (a) simply connected (b) connected but not simply connected, (c) disconnected.*

Analogously to the corresponding topological classes of deformations, for a sharp or finished hole the trimmed area does not intersect the surface boundary, differently from inlets, while gaps can be either internal or crossing the surface boundary. On this regard, introducing also for gaps the term *isle* ($\tau - isle$) for each possible connected component whose boundary is internal to another closed curve of the influence boundary, similarly to channel deformations, we distinguish several types of gaps:

**Def. 20** *A gap cut, giving raise to $n$ $\tau$-isles, is said n-gap, for $n = 0, 1, 2 \ldots$.*

A 0-gap always intersects the surface boundary. A 1-gap is topologically equivalent to a removed ring from a surface. A typical use of the gap feature is for separating the different components composing the object from the overall surface. Fig. 15 shows a surface containing a hole, an inlet and $n$-gaps for $n = 0, 1, 2$. See also Fig. 16 that shows details of objects containing a 0-gap, a hole and an inlet respectively.

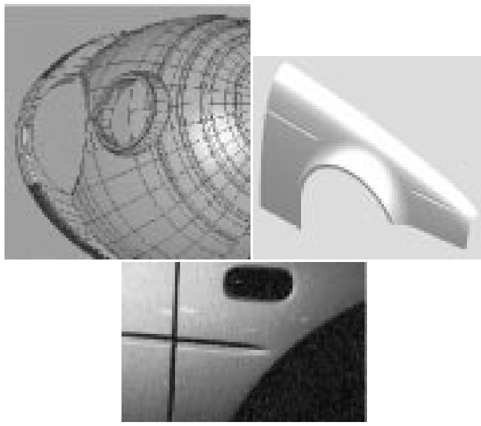

**Figure 16:** *Examples of cuts*

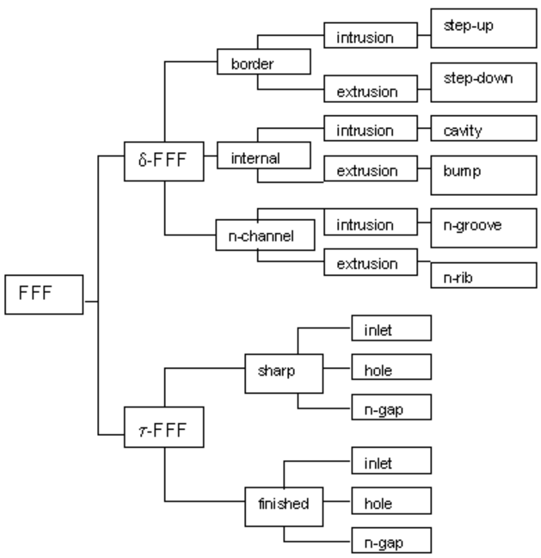


**Figure 17:** *FFF classification*

## 8. FFF defining parameters

In Fig. 17 a schematic diagram is given, resuming the classification of free form features defined in the previous sections.

This classification is a preliminary activity done in order to specify features useable for creating complex shapes. For doing this, it is necessary to provide a precise definition of the deformation functions and elimination laws possibly through the use of a set of parameters meaningful for the user. As previously stated, in aesthetic design detail features are particularly used to emphasise the effect of special lines, the so-called character lines, which usually do not correspond to real curves on the surfaces but to well perceived curves. Such curves can be the maximal deformation under a specific point of view, inflection lines or curves with high curvature properties. Using a deformation function which allows the direct manipulation of points and curves on a surface, as the one developed in FIORES [20], it is then possible to identify a set of significant parameters to drive the deformation [16]. Particular conditions and restrictions on them depend on the considered feature class. Their complete specification will be topic of a future paper, here we simply list the possible categories and briefly describe their meaning:

- *Influence area boundary*. It can be specified through the explicit definition of the curves composing it or automatically derived from the specification of the designed character line, according to special rules depending on the feature types.
- *Transition conditions* along the influence boundary specifying the type of continuity to be satisfied among the transition surface and the unmodified surface area (e.g. $G^{-1}, G^0, G^1, G^2$ conditions). Different continuity conditions could be specified on different portions of the boundary.
- *Leading lines*, which are used to specify how particular curves on the domain are modified by the deformation function (e.g. character lines, inflection curves).
- *Leading points*, which are used to specify the value of the deformation applied at the point.
- Boundary of *specific characteristic regions*. They depend on the feature type (trim area for cuts, displacement areas, etc.).

The parameters we have just mentioned give the basic structure to describe a lot of local surface deformations and eliminations representing detail features. In addition, the designer could have the necessity to specify also some other parameters in order to characterize the shape of the modified surface for a better control of the feature shape. Some of them are here suggested:

- *Sections* along given direction(s).
- *Curvature and gradient laws* along specified curves or points.

It is worth observing that the classification proposed is also aimed at automatically deriving some of the parameter values depending on the specified feature type. Moreover, further checks have to be made in order to ensure consistency among the various involved parameters.

## 9. Conclusions and future work

In this paper a formal classification of simple free form features has been proposed. With regard to this, Fig. 18 shows significant examples of different simple detail features characterizing the dashboard of a car. The suggested classifica-

**Figure 18:** *Several types of free form features*

tion is based on the topological and morphological characteristics associated to the deformation functions used to create the features. In particular two main classes of features have been identified: those corresponding to elastic deformations and those considering elimination of surface regions. The work is aimed at allowing the identification of a useful set of user friendly parameters, possibly having an aesthetic meaning, and conditions, which can be hard-coded, to define meaningful deformation having predictable behaviour.

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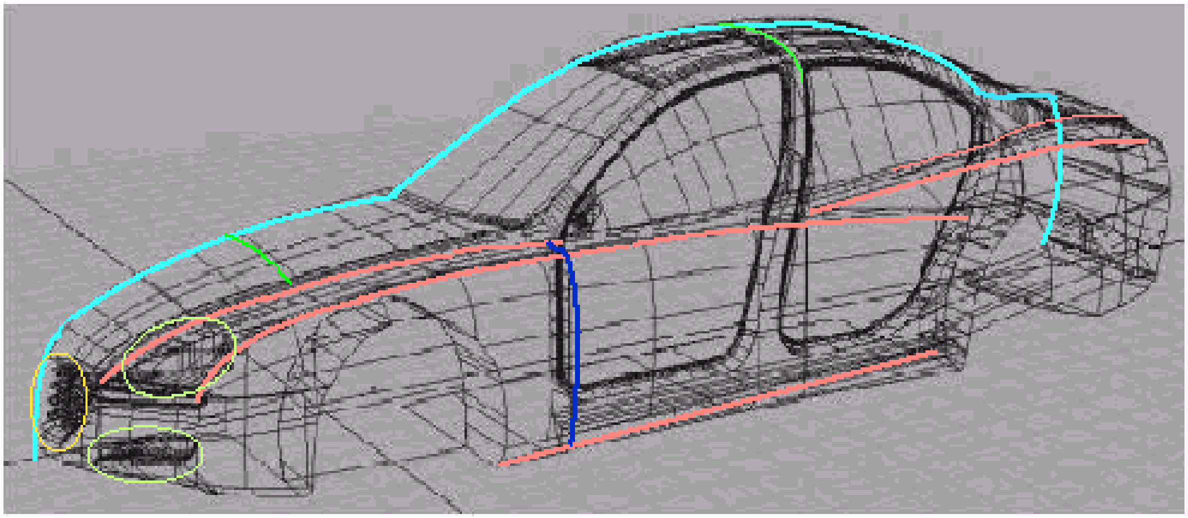

**Figure 19:** *[Fontana et al.] Nautilus (by PininFarina)*